THE AMERICAN MATHEMATICAL MONTHLY.

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BIOGRAPHY.

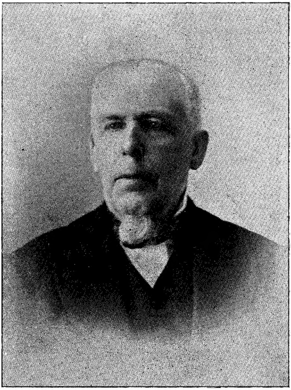
DANIEL KIRKWOOD.

BY ROBERT J. ALEY, A. M., PROFESSOR OF MATHEMATICS IN THE INDIANA STATE UNIVERSITY, BLOOMINGTON, INDIANA.

"His is truly a great name in science, with a world wide renown." So wrote Professor P. Piazzi Smyth, Astronomer Royal for Scotland in 1885. He who knows Dr. Kirkwood either personally or through his contributions to science, gladly gives assent to the Astronomer Royal's eulogy. The Indiana University, whose faculty he for so many years honored, has just named in his honor the large handsome new building now being constructed. This is her first building named after a living man.

Daniel Kirkwood is of Scotch Irish descent, his grandfather coming from Ireland and settling in Delaware in 1771. His parents were both born in this country. He was born in Hartford County, Maryland, September 27,1814. He had only the usual advantages of a farmer boy in those days. Not particularly liking the life of a farmer he turned his attention to teaching and at the age of 19, he took charge of a country school at Hopewell, York County, Pennsylvania. In this school a young man desired to study Algebra. A copy of Bonnycastle was secured and together teacher and student explored its mysteries. This year's work aroused his interest in mathematics and no doubt had much to do in shaping his future.

In 1834 he entered York County Academy at York, Pennsylvania. His work here must have been of a superior kind, for in 1838 he was elected first assistant and mathematical instructor. He held this position until 1843 when he became principal of the Lancaster, Pennsylvania High School. While here he married Miss Sarah A. McNair of Newton, Pennsylvania. In 1851 he became professor of mathematics in Delaware College. In 1854 he was pro-



Daniel Kirkwood.

moted to its presidency. At the expiration of two years he resigned to accept the chair of mathematics in Indiana University. With the exception of one year 1866-7 when he was professor of mathematics in Washington and Jefferson College, he held the chair of mathematics in Indiana University until 1886, when he resigned on account of failing health. He then became Emeritus Professor, a position which he still holds.

In 1889 he removed to Riverside, California. At the opening of Leland Stanford Jr. University he was appointed Non Resident Lecturer on Astronomy. He is now in his eightieth year and is passing his declining years pleasantly with a favorite nephew on an orange ranch in Southern California.

His natural bent for mathematics found its activity in application to astronomical problems. His whole life has been devoted to theoretical and mathematical astronomy. Never having access to an observatory himself he has been content to take the observations of others and from them work out those broad generalizations and those specific explanations that have been of such great value to astronomical science. In his case there is scarcely a doubt but that the lack of an observatory has been a real benefit to Astronomy. His peculiar strength lay in the line of theoretical astronomy and in this line his work has been done. An observatory might have turned him aside. He is a member of the American Philosophical Society and of the American Association for the Advancement of Science. To these two societies many of his most valuable papers were first read.

In 1849 he made public what is now known in Astronomical literature as "Kirkwood's Law." This at once gave him prominence. Because of this descovery, Proctor has named him the "Kepler of America." As this law has not yet found its way into many popular astronomies it is quoted here entire:

KIRKWOOD'S LAW.

"Let P be the point of equal attraction between any planet and the one next interior, the two being in conjunction; P' that between the same and the one next exterior.

Let also $D=$ the sum of the distance of the points P, P' from the orbit of the planet; which I shall call the diameter of the sphere of the planet's attraction;

$D'=$the diameter of any other planet's sphere of attraction found in like manner;

$n=$the number of sidereal rotations performed by the former during one sidereal revolution around the sun;

$n'=$the number performed by the latter; then it will be found that,

$n^2:n'^2: :D^3:D'^3$; or $n=n'\left(\frac{D}{D'}\right)^{\frac{3}{2}}$.

That is, the square of the number of rotations made by a planet during one revolution around the sun, is proportional to the cube of the diameter of its sphere of attraction; or, $\frac{n}{D^{\frac{3}{2}}}$ is a constant quantity for all the planets of the Solar System."

This law was subjected to a rigid mathematical examination by Sears C. Walker in the American Journal of Science, New Series, vol. X. pp. 19-26. Dr. B. A. Gould, Jr. in the same number of the American Journal of Science shows how the Law supports the Nebular Hypothesis.

The Law as originally stated has been subjected to slight modifications by the author in recent years. It has never been mathematically demonstrated. It is yet in the same condition that Kepler's Laws were when they were first announced.

Dr. Kirkwood has given much attention and study to the subject of Comets and Meteors. In this field he is an authority. His two books, "Meteoric Astronomy" 1867, and "Comets and Meteors" 1873 are both well known. Miss Clerke in her History of Astronomy, p. 381, in speaking of Coments and Meteors says:

"Professor Kirkwood, however, by a luminous intuition, penetrated the secret so far as it has been yet made known. In an article published, in the Danville Quarterly Review for December 1861, he argued from the observed division of Biela, and other less noted instances of the same kind, that the sun exercises a "divellent influence" on the nuclei of comets, which may be presumed to continue its action until their corporate existence (so to speak) ends in complete pulverization. "May not,' he continued, 'our periodic meteors be the debris of ancient, but now disintegrated comets, whose matter has become distributed round their orbits."

Many of his contributions to current scientific literature relate to comets and meteors. His study on these subjects has done much to verify and slightly modify the Nebular Hypothesis.

When about fifty asteroids were known Dr. Kirkwood announced the theory that in those spaces were simple commensurability of motion with that of Jupiter occurs, there must be gaps in the asteroid zone. The theory was based on mathematical and physical facts. It was at once received with favor and in 1870 Proctor spoke of it in the highest terms. At present the large number of known asteroids goes far to verify the theory. There is scarcely a doubt that the physical facts underlying the law of commensurability have in the main regulated the distribution of the asteroids. Only two or three exceptions, tne most prominent of which is the minor planet Menippe, are yet known. Dr. Kirkwood applied the same theory to the rings of Saturn and found that the breaks in the rings occured just where commensurability of motion with Saturn's satellites would indicate they should be. Dr. Meyer of Geneva in a work on Saturn's Rings has worked out in detail the theory suggested by Kirkwood. Kirkwood calls attention to this in a cummunication to the American Philosophical Society, and makes clear his claim to priority. He also expresses great gratification that so distinguished an authority as Dr. Meyer should verify his theory.

His life has been a very busy one. Its fifty active years have been spent in teaching, and his scientific contributions have been made in addition to the duties of the school room, which were never in the least slighted. The great

bulk of his writings appear in the Proceedings of the American Philosophical Society. The Sidereal Messenger and the American Journal of Science and Arts. But as his bibliography will show, many other scientific and literary journals have been honored by contributions from his pen.

In his adopted State, Indiana, he is held in the highest esteem. The state Teachers' Association in 1859 elected him mathematical editor of the Indiana School Journal. The mathematical department of the Journal under his care was very strong. Besides giving it his careful direction he contributed many notes, comments and solutions. After nearly four year's service he resigned because of lack of time to give to it the attention demanded.

For perhaps fifteen years he was a regular contributor to "The Journal" of Indianapolis, a leading daily of the State. All the current astronomical events were duly written up. Many of the articles appeared unsigned on the editorial page, while many more appeared on the literary page duly signed. It is safe to say that while Daniel Kirkwood corresponded for the Journal no daily in the country surpassed it in the trustworthiness of its astronomical statements.

Prior to 1885 he was employed by the Appleton's to write the articles on Astronomical Progress for their Annual Encyclopaedia. These articles,as indeed is true of all his writings, are characterized by brevity, clearness and accuracy. Few men have possessed in a higher degree than he the ability to say so much in so few words. His most remarkable theories and conclusions have been conveyed to the public in articles remarkable both for their simplicity and brevity.

To write the plain truth about the personal character of Daniel Kirkwood is to write such an eulogy as most men give to their ideal hero. By his pupils he is universally loved. The admiration, almost reverence they have for him is admirably illustrated in this statement made by one of his students years ago, "When I die I want to go where Dr. Kirkwood goes." The personal charm of his character is manifest to those who know him only through science and correspondence. Some years ago when Proctor was making a lecture tour of the United States, he lectured in Indianapolis. After the lecture he was approached by a delegation from Greencastle requesting him to lecture there the next evening. He said, "no I cannot do so. I came from England to America to see Daniel Kirkwood. To morrow is my opportunity and I am going to Bloomington to see him."

The writer well remembers his first visit to Bloomington. He went into a barber shop and as it was a rainy day there was quite a crowd of loafers, white and black, professional men and day laborers. By chance the conversation turned to men. Every man present found his ideal in Daniel Kirkwood. No man ever received a higher tribute of praise. His life is so simple, so pure, and so true that the student, the philosopher and the common man can all find in him the ideal. So sweet and well tempered is his disposition that if he has or ever had an enemy no one knows it.

In religion he is a Presbyterian. Although a strong believer in the

Westminster Confession, his broad mind has charity for all who try in any way to follow in the footsteps of the blessed Jesus. His study of the stars has but strengthened his belief in God. To his mind, with its faith strengthened by an almost infinite grasp of the mightiest works of God, unbelief is impossible and he can hardly understand how honest unbelief can exist in another. Some years ago he gave a beautiful demonstration before a class. A student asked "is that always true?" "yes" said he, "as true as that there is a God in Heaven." "But," said the student, "what would you say to him who does not believe in God?" Straightening up to his full height, and with glittering eye he said, "I would try to keep my temper and get away as quickly as possible."

As a teacher he has come in contact with thousands of young men and women. He is not a believer in educational forcing and so the student who did not wish to learn could get through his work without very great effort; but the earnest student found in him a help and an inspiration. Although gifted with unusual mathematical ability he appreciates the difficulties that confront the young learner. His remarkable ability in explaining a difficulty by a few words gave his classes unbounded confidence in him. Neatness and accuracy in thought and expression he constantly demanded. But perhaps the greatest lesson he taught was that of his simple, sweet life. A well known alumnus of Indiana University said in a public lecture a few years ago, "The specific lessons of the class room, the formulae and theorems of my college course have been forgotten, but there is one thing worth more than all else that will ever abide, the lesson in true life given me by my daily contact with the noble astronomer, Daniel Kirkwood."

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Note on 279th Asteroid.............................Vol. VIII.
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On the Nebular Hypothesis and the Approximate Commensurability of the Planetary Periods of............Vol. 29.
Relations Between the Motions of Some of the Minor Planets...................................Vol. 35.

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED, A. M., (Princeton); Ph.D., (Johns Hopkins); Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

CHAPTER SECOND.

THE FIRST TREATISE ON NON-EUCLIDEAN GEOMETRY.

[Continued from the April Number.]

Is it not surprising that a book so remarkable, that it will henceforth forever mark an epoch in human thought, should have been forgotten for more than a century and a half? The first treatise on Non-Euclidean Geometry, a

work of extraordinary genius, appeared in 1733, yet so far as I know, not one single copy of this wonderful book is owned on the Western Continent.

That I have had the book for a considerable time in my possession is due to the generosity of a learned prelate of Louvain, R. P. Thirion, S. J., who lent me, across the ocean, his only copy of this rare and inestimable treasure.

I have already made the distinction between Anti-Euclideans, such as Bertrand of Geneva, Legendre, M. Vincent, Ed. T. Dixon, etc., who try to convict Euclid of imperfection by offering short proofs of his celebrated Parallel-Postulate; and the true Non-Euclideans, such as Lambert, Bolyai, Lobatschewsky, Riemann, Helmholtz, Lie, Klein, Clifford, Cayley, etc., all ardent admirers of Euclid, but makers of two companion geometries, called usually Lobatschewsky's and Riemann's.

But a whole century before Lobatschewsky, it must have been by a sort of prophetic instinct that the Italian Saccheri called his book, now ressurrected for immortality, "Euclid vindicated from every fleck." The complete title is as follows: Euclides—ab omni naevo vindicatus:—sive—conatus geometricus—quo stabiliuntur—Prima ipsa universae Geometriae Principia.—Auctore—Hieronymo Saccherio—Societatis Jesu—In Ticinensi Universitate Matheseos Professore.—Opusculum—Ex.^{mo} Senatin—Mediolanensi—Ab Auctore Dicatum.—Mediolani, MDCCXXXIII.—Ex Typographia Pauli Antonii Montani. Superiorum permissu.

This book is an intensely interesting historical study, not only in science, but in psychology, and ethics; for it is evident to any mathematician reading it with the light we now have, that Saccheri deliberatly built up a non-Euclidean geometry, and then covered it with just enough disguise to enable it to pass the the ordeal of authorization by Ignatius Vicecomes, Provincial of the Jesuits, who certefies that it had been read by some Theologians of that society and found fit to see the light; and to enable it to receive the "*Imprimatur*" of the Senate, a Cardinal, and the Inquisitor General, Sylvester Martini, by whose order it was carefully read by the Revisor Don Gaspar, Doctor both of law and of Sacred Theology, who declared it to contain nothing against the orthodox faith.

Remember DeMorgan's saying: "As to writing another work on geometry, the middle ages would as soon have thought of composing another New Testament......his order of demonstration was thought to be necessary and founded in the nature of our minds;" and remember that Saccheri's book contains not merely "another *work* on geometry," but *another geometry*, a thought so tremendous, so unorthodox, that its discovery in his book by these great Church Dignitaries would have doomed Saccheri to death. Perhaps an after suspicion of the truth *did* doom him to death, for the permission of the Provincial was given August 16th, 1733, and Saccheri was dead October 25th, 1733.

There exists in the library of Modena, Italy, a Manuscript Biography of Saccheri. Dr. Emory McClintock, the able President of the New York

Mathematical Society, writes of Saccheri: "He confessed to a distracting heretical tendency on his part in favor of the 'hypothesis anguli acuti,' a tendency against which, however, he kept up a perpetual struggle. After yielding so far as to work out an accurate theory anticipating Lobatschewsky's doctrine of the parallel-angle, he appears to have conquered the internal enemy abruptly, since, to the surprise of his commentator Beltrami, he proceeded to announce dogmatically that the specious 'hypothesis anguli acuti' is positively false." Of course no such confession occurs or could have occured in the book itself; for with it the book could never have been printed. If such a statement occurs in the Manuscript Biography that must have been the work of a personal friend written after Saccheri's death.

The sudden dogmatic assertion above mentioned does occur, first on page 70 of the work, a quarto. But this, after seventy quarto pages of rigorous logic and elegant demonstration to establish a non-Euclidean geometry, may be looked upon as something like the stucco for the king's inspection with which the immortal architect in Egypt covered the stone bearing his own name.

A whole century later, 1829, Gauss, in the "Lehr-und Lernfreiheit" of a German University, writes to his friend Bessel that he will never publish his researches on this subject, "da ich das Geschrei der Gegner scheue, wenn ich meine Ansicht ganz aussprechen wollte."

Saccheri's marvellous book says all it could have said and existed; apart from any decision of the historical question as to how consciously the Italian jesuit was in it practising the motto of his order: "The end justifies the mean's.

The expression on the title-page, "the very First Principles of Universal Geometry," strongly suggests Lobatschewsky's *Pangeometry*."

The book opens with a dedication to the Senate of Milan, followed by a Preface to the Reader.

This, after a powerful eulogy of Euclid, begins by stating that the Parallel-Postulate has ever been a vexed question. "Though heretofore no one has doubted the truth of the proposition, yet many have maintained that it is not axiomatic. So not a few have attempted to demonstrate it from those propositions of the First Book of Euclid which precede the twenty-ninth, wherein it is first used. But since all these attempts have signally failed, many, more recently, have attacked the matter by setting up a certain new definition of parallels.

Whereas Euclid defines parallels thus 'Any straight lines, which are in the same plane, and being produced indefinitely towards both sides meet each other on neither, are parallels;' these, for the latter words substitute '*are always mutually equidistant*.' But here arises a new split.

For some, and those certainly the more acute, have endeavored to show the existence of parallel straight lines as thus defined, whence they make the transition to the proposition to be demonstrated under Euclid's own words.

But others (not without a great sin against rigorous logic) assume as given such parallel straight lines, forsooth *equidistant*, that thence they may make the step to what remains to be proved." Saccheri tnen goes on to say that he will not at first go into the question of the nature of those lines equidis-

tant at all points from a certain line supposed straight, but will return to it later. "But any one can see that herein is occasion for subjecting to a rigorous examination the very first principles of universal geometry."

This is Saccheri's excuse, his plea, his defense for introducing a new kind of geometry.

"Atque hinc incipit diuturnum praelium adversus hypothesin anguli acuti."

ARITHMETIC.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

13. Proposed by J. R. BALDWIN, A. M., Professor of Mathematics in the Davenport Business College, Davenport, Iowa.

A man borrowed \$5000 at a western bank giving his note for \$5000 due in 5 years without grace at 8% interest payable annually, and pays the banker a bonus of \$500 in cash for making the loan; what rate per cent. does he pay?

I. Solution by ROBERT J. ALEY, A. M., Professor of Mathematics in the Indiana University, Bloomington, Indiana.

Simple interest on \$5000 for 5 years at 8% is \$2000.

If the interest is paid annually in advance, the loss of the use of the money to the borrower is the interest on \$400 for $(5+4+3+2+1)$ years which is \$480. If the interest is paid annually but not in advance the loss is the interest on \$400 for $(4+3+2+1)$ years which is \$320. Hence the total interest paid is

$$\$2000+\$500+\$480=\$2980$$
$$\text{or } \$2000+\$500+\$320=\$2820.$$

Interest for 1 year is $\frac{1}{5}$ of \$2980=\$596, or $\frac{1}{5}$ of \$2820=\$564.

$$\$596\div\$5000=11.92\%$$
$$\$564\div\$5000=11.28\%.$$

II. Solution by FRANK HORN, Kidder Institute, Kidder, Missouri.

II.
1. 100% = \$5000 = face of note.
2. 1% = \$50.
3. 8% = $8\times$\$50 = \$400 = interest for 1 year.
4. $5\times$\$400 = \$2000 = interest for 5 years.
5. \$2500 = \$2000 + \$500 = interest paid including the bonus.
6. \$4500 = \$5000 − \$500 = amount the borrower kept.
7. ∴ \$2500 = the interest on \$4500 for 5 years.
8. \$500 = $\frac{1}{5}$ of \$2500 = the interest for 1 year.
9. \$4500 = 100% of itself.
10. \$1 = $\frac{1}{45}$%.
11. \$500 = $500\times\frac{1}{45}\% = 11\frac{1}{9}\%$.

III. $\therefore 11\frac{1}{9}\%$ = rate of interest paid.

Solved with varying results by *M. A. Gruber*, *G. B. M. Zerr*, *J. R. Baldwin*, *H. C. Whitaker*, *H. W. Draughon*, *I. L. Beverage*, and *W. F. Bradbury*. Some of the contributors used compound interest.

14. Proposed by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

A bank by discounting a note of 7% receives for its money a discount equivalent to $7\frac{3}{4}\%$ interest. How long must the note have been discounted before it was due?

Solution by the Proposer.

$7\frac{3}{4}\%-7\%=\frac{3}{4}\%$. $7\frac{3}{4} : \frac{3}{4}=\$1.00 : \$\frac{3}{31}$, the interest of \$1.00 for the required time at 7%.

$\$\frac{7}{100} : \$\frac{3}{31}=12$ months, : $16\frac{128}{217}$ months.

$\therefore$ Time $=16\frac{128}{217}$ months $=1$ year 4 month $17\frac{151}{217}$ days.

Also solved with different results by *H. C. Whitaker* and *P. S. Berg*.

15. Proposed by O. S. KIBLER, Superintendent of Schools, West Middleburg, Logan County, Ohio.

Supposing the town A to be 30 mi. from B, B 25 mi. from C, C 20 mi. from A, where must a house be erected to be equally distant from each of the towns?

Solution by W. A. GRUBER, War Department, Washington, D. C.

The radius of the circumscribed circle of the triangle formed by drawing AB, BC, and AC, is the *distance* required, and the center of this circle is the *place* for the erection of the house.

From Geometry or Trigonometry, we get

$$R=\frac{abc}{4\sqrt{s(s-a)(s-b)(s-c)}},$$

in which R represents radius of circumscribed circie of a triangle in terms of the sides of triangle.

Substituting for a, b, c, and s $[=\frac{1}{2}(a+b+c)]$, the respective values 25, 20, 30, and $37\frac{1}{2}$, we have

$$R=\frac{25\times20\times30}{4\sqrt{37\frac{1}{2}\times12\frac{1}{2}\times17\frac{1}{2}\times7\frac{1}{2}}}, \text{ which reduced, becomes}$$

$$R=\frac{40}{\sqrt{7}}=\frac{40\sqrt{7}}{7}=15.11857 \text{ mi.}$$

Also solved by *H. C. Whitaker*, *G. B. M. Zerr*, *Seth Pratt*, *J. F. W. Scheffer*, *J. W. Watson*, and *P. S. Berg*.

PROBLEMS.

22. Proposed by E. S. Loomis, A. M., Ph.D., Professor of Mathematics, Baldwin University, Berea, Ohio.

A borrows \$1000 from B for 10 years, on which he pays 4% semi-annually.

A immediately loans the \$1000 to C for 10 years, who agrees to pay to A \$$12\frac{1}{2}$ on the first of each month for 120 mos. or 10 yrs., at which time the whole debt is considered canceled, C no longer being, in any way, indebted to A. Upon the receipt of each of the \$$12\frac{1}{2}$ payments made by C, A immediately reloans it to D, E, F, etc., upon the same conditions as he loaned the \$1000 to C; at the end of 120 mos. all who are indebted to A pay up in full all due him, and he (A) pays B the principal, all interst having been paid when due.

Query: How many dollars has he in hand?

23. Proposed by H. C. Whitaker, Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.

A rectangular hall 80 feet long, 40 feet wide and 12 feet high has a spider in one corner of the ceiling. How long will it take the spider to crawl to the opposite corner on the floor if he crawls a foot in a second on the wall and two feet in a second on the floor?

24. Proposed by Mrs. Mary E. Hogsett, Danville, Kentucky.

On January 4, 1889, it was noticed that a clock was 15 minutes fast. On March 1, 1894, it was found to be six and one half minutes slow. When and what time was accurate time?

Solutions to these problems should be received on or before July 1st.

ALGEBRA.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

11. Proposed by ISAAC L. BEVERAGE, Monterey, Virginia.

Two men, A and B, had a money-box, containing \$210, from which each drew a certain sum daily; this sum being fixed for each, but different for the two. After six weeks, the box was empty. Find the sum which each man drew daily from the box, knowing that A *alone* would have emptied it five weeks earlier than B *alone*,

I. Solution by W. L. HARVEY, Portland, Maine.

Let $x=$ amount B drew out daily, $mx=$ amount A drew out daily.

Then, $\dfrac{210}{x}-\dfrac{210}{mx}=35\ldots(1)$, and $\dfrac{210}{x(m+1)}=42\ldots(2)$.

Solving $m=1\frac{1}{2}$, whence $x=\$2$, and $mx=\$3$.

II. Solution by P. S. BERG, Apple Creek, Ohio.

Let $x=$ what A drew out daily, $y=$ what B drew out daily.

Then, $42x+42y=\$210\ldots(1)$, and $\dfrac{210}{y}-\dfrac{210}{x}=35\ldots(2)$.

Whence $x=3$, and $y=2$. $\therefore$ A drew out daily \$3, and B \$2.

Also solved by *M. A. GRUBER, H. W. DRAUGHON, H. C. WHITAKER, C. E. MYERS, G. B. M. ZERR, A. L. FOOTE*, and *ROBERT J. ALEY*.

12. Proposed by F. M. SHIELDS, Coopwood, Mississippi.

Three Lads, A, B, and C, each climbed to the top of an upright pole: A's pole was 20 feet high, B's 60 feet, and C's pole was 100 feet high. They all started at the same time, and each climbed up a part of the way, at the same rate of speed per minute, and after each rested five minutes, they ascended to the tops of their respective poles, at the same rate of speed per minute, when they found that each had consumed

the same length of time, 25 minutes each, (including the 5 minutes each rested on the way). How far up did each climb before resting? At what rate of speed per minute did each climb?

Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

The time of ascent *before* resting *plus* the five minutes' rest *plus* the time of ascent *after* resting $=25$ minutes. $\therefore$ The time of ascent *before* resting *plus* the time of ascent *after* resting $=20$ minutes.

Let x=rate of speed per minute *before* resting,
y= " " " " " *after* " ;
m=time, in minutes, of A's ascent *before* resting,
n= " " " " B's " " " ,
p= " " " " C's " " " ;
$20-m$= " " " " A's " *after* " ,
$20-n$= " " " " B's " " " ,
$20-p$= " " " " C's " " " .

$mx+(20-m)y=20$, or $m(x-y)=20-20y$, (1).
$nx+(20-n)y=60$, or $n(x-y)=60-20y$, (2).
$px+(20-p)y=100$, or $p(x-y)=100-20y$, (3).

Subtracting (1) from (2), and (2) from (3),

$(n-m)(x-y)=40$, or $x-y=\dfrac{40}{n-m}$, (4)

$(p-n)(x-y)=40$, or $x-y=\dfrac{40}{p-n}$, (5).

From (4) and (5),

$\dfrac{40}{n-m}=\dfrac{40}{p-n}$, or $p=2n-m$, (6)

Adding (1) and (2),

$(n+m)(x-y)=80-40y$, or $x-y=\dfrac{40(2-y)}{n+m}$, (7).

From (4) and (7),

$\dfrac{40(2-y)}{n+m}=\dfrac{40}{n-m}$, or $2-y=\dfrac{n+m}{n-m}$, or $y=2-\dfrac{n+m}{n-m}$, (8).

From (8) we observe that when $\dfrac{n+m}{n-m}=2$, $y=0$; when $\dfrac{n+m}{n-m}>2$, y is *negative*; and when $\dfrac{n+m}{n-m}<2$, y is *positive*.

Put $n=m+c$; then $\dfrac{n+m}{n-m}=\dfrac{2m+c}{c}$, $y=\dfrac{c-2m}{c}=1-\dfrac{2m}{c}$, $p=2n-m$ $=m+2c$, and, from (4), $x=\dfrac{40}{n-m}+y=\dfrac{40}{c}+y$, or $x=\dfrac{40-2m}{c}+1$.

Since, for positive values of y, $\dfrac{n+m}{n-m}<2$, then $\dfrac{2m+c}{c}<2$; whence $c>2m$. We also find $n>3m$, and $p>5m$.

As $c>2m$, y is always a proper fraction, except when $m=0$; then $y=1$.

20 minutes is the greatest amount of time that can be consumed in actual ascent. It will readily be seen that p *alone* may equal 20, and that 20 is the greatest value p can have for *positive* values of all times of ascent.

Substituting 20 for p in $p=m+2c$ and $p>5m$, we have $m+2c=20$ and $5m<20$; whence $c=10-\frac{m}{2}$ and $m<4$.

It is, therefore, seen that the limits for m are $m=0$ and $m<4$, and that c is found between $c>2m$ and $c=10-\frac{m}{2}$. By putting $m=0$, the limits of c are found to be $c>0$ and $c=10$.

The conditions of the problem have each of the lads ascend *part* of the way *before* resting and the remainder of the way *after* resting. The values $m=0$, $p=20$, and $c=10$, do, therefore, not apply to this problem. According to the conditions of the problem the limits of m are $m>0$ and $m<4$, and the limits of c are $c>0$ and $c<10$.

When $m=1$, c may have any value between $c>2$ and $c=9\frac{1}{2}$,
When $m=2$, c .. .. $c>4$ and $c=9$,
When $m=3$, c .. .. $c>6$ and $c=8\frac{1}{2}$,
When $m=\frac{1}{2}$, c .. .. $c>1$ and $c=9\frac{3}{4}$,
When $m=1\frac{1}{2}$, c .. .. $c>3$ and $c=9\frac{1}{4}$, &c., &c.

We accordingly observe that there may be many *sets* of *values* for the distances climbed and rates of speed; the number of *sets* and the *values* being restricted only by the limits of m and c.

To find the distances climbed by each before and after resting and the rates of speed, we have the following formulae and conditions:

m may have any value between $m>0$ and $m<4$.

c may have any value between $c>2m$ and $c=10-\frac{m}{2}$, the value of c depending on m, except that we know that the limits of c are $c>0$ and $c<10$.

$n=m+c$, $p=m+2c$, $y=1-\frac{2m}{c}$, and $x=\frac{40}{c}+y$ or $\frac{40-2m}{c}+1$; and mx, nx, and px are respectively the distances which A, B, and C ascend *before* resting.

We will now solve for a few sets of values.

Put $m=1$; then $c=$ any value between $c>2$ and $c=9\frac{1}{2}$.

Put $c=3$; then $n=4$, $p=7$,
$x=13\frac{2}{3}$ and $y=\frac{1}{3}$;
$\therefore$ $mx=13\frac{2}{3}$, $nx=54\frac{2}{3}$, and $px=95\frac{2}{3}$

Put $c=4$; then $n=5$, $p=9$
$x=10\frac{1}{2}$ and $y=\frac{1}{2}$;
$\therefore$ $mx=10\frac{1}{2}$, $nx=52\frac{1}{2}$, and $px=94\frac{1}{2}$.

Put $c=8$; then $n=9$, $p=17$,
$x=5\frac{3}{4}$, and $y=\frac{3}{4}$;
$\therefore$ $xm=5\frac{3}{4}$, $nx=51\frac{3}{4}$, and $px=97\frac{3}{4}$.

Put $m=2$; then c=any value between $c>4$ and $c=9$.

Put $c=6$; then $n=8$, $p=14$,

$x=7$ and $y=\frac{1}{3}$;

$\therefore\ mx=14$, $nx=56$, and $px=98$.

Put $c=8$;. then $n=10$, $p=18$,

$x=5\frac{1}{2}$ and $y=\frac{1}{2}$;

$\therefore\ mx=11$, $nx=55$, and $px=99$.

Put $c=7\frac{1}{2}$; then $n=9\frac{1}{2}$, $p=17$,

$x=5\frac{4}{5}$ and $y=\frac{7}{15}$;

$\therefore\ mx=11\frac{3}{5}$, $nx=55\frac{1}{10}$, and $px=98\frac{3}{5}$.

Put $m=3$; then c= any value between $c>6$ and $c=8\frac{1}{2}$.

Put $c=7$; then $n=10$, $p=17$,

$x=5\frac{6}{7}$ and $y=\frac{1}{7}$;

$\therefore\ mx=17\frac{4}{7}$, $nx=58\frac{4}{7}$, and $px=99\frac{4}{7}$.

Put $m=\frac{1}{2}$; then c=any value between $c>1$ and $c=9\frac{3}{4}$.

Put $c=2$; then $n=2\frac{1}{2}$, $p=4\frac{1}{2}$,

$x=20\frac{1}{2}$ and $y=\frac{1}{2}$;

$\therefore\ mx=10\frac{1}{4}$, $nx=51\frac{1}{4}$, and $px=92\frac{1}{4}$.

Put $c=8$; then $n=8\frac{1}{2}$, $p=16\frac{1}{2}$,

$x=5\frac{7}{8}$ and $y=\frac{7}{8}$;

$\therefore\ mx=2\frac{15}{16}$, $nx=49\frac{15}{16}$, and $px=96\frac{15}{16}$.

Hence, by assigning to m any value found within its limits, and finding the limits of c with reference to the respective values of m, the *sets of values* for x, y, mx, nx, and px are almost numberless. It will, however, be observed that all the values of m, except 1, 2, and 3,are fractional.

Another method of solution is the use of a *multiplier* instead of a *difference* in the relations of m, n, and p.

Put $n=am$; then $p=m(2a-1)$, $\dfrac{n+m}{n-m}=\dfrac{a+1}{a-1}$, $y=\dfrac{a-3}{a-1}$, and

$x=\dfrac{40}{m(a-1)}+y$.

As previously stated, $\dfrac{n+m}{n-m}<2$; therefore $\dfrac{a+1}{a-1}$ <2; whence $a>3$.

Substituting 20 for p in $p=m(2a-1)$, we find $a=\dfrac{10}{m}+\dfrac{1}{2}$.

Hence the limits for a are $a>3$ and $a=\dfrac{10}{m}+\dfrac{1}{2}$.

To show the process of work, we will solve for a few *sets of values*.

Put $m=1$; then a=any value between $a>3$ and $a=10\frac{1}{2}$.
Put $a=4$; then $n=4, p=7$,
$x=13\frac{2}{3}$ ane $y=\frac{1}{3}$;
$\therefore\ mx=13\frac{2}{3}$, $nx=54\frac{2}{3}$, and $px=95\frac{2}{3}$.

Put $m=2$; then a=any value between $a>3$ and $a=5\frac{1}{2}$.
Put $a=4$; then $n=8$, $p=14$,
$x=7$ and $y=\frac{1}{3}$;
$\therefore\ mx=14$, $nx=56$, and $px=98$.

For want of space, we will let the reader solve for sets of values when $m=0$, $m>4$, and $a<3$. The discussion of these values may prove interesting.

Also solved by *H. W. DRAUGHON.*

13. Proposed by H. C. WHITAKER, B. S., M. E., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.

Six city boys, Jim, Josh, Jerry, Jack, Jake and Jeorje went into the country to steal apples from a tree. While three kept watch, the other three climbed up and got what they wanted. Then they came down while the other three rascals went up and stole. The one that got most was one of the last to go up.

Each trio of theives took the same number and had each boy taken as many as he did take in *each of that number of pockets*, each trio would also have taken the same number and the tree would have lost 538 apples. As it was, Josh got more than Jack, but Jeorje got as many as Josh and Jack together, while Jake got twice as many as Jerry and two more than Jim. What were the names of the three that first kept watch?

Solution by H. W. DRAUGHON, Clinton, Louisana.

Let x= Jack's share, y= Josh's share and z=Jerry's share. Then from the conditions, $x+y$=Jeorje's share, $2z$=Jake's share, and $2z-2$=Jim's share. To find each share in integers, we must separate $\frac{1}{2}\times538=269$,—the numbers each trio would have taken,had the number in each share been squared, into 2 sets of 3 square numbers the sum of the roots in the two sets being equal. We easily find the required numbers to be, $(13)^2$, $(8)^2$, $(6)^2$, and $(12)^2$, $(10)^2$, $(5)^2$. Since the greatest root is 13, the boys who took respectively 13, 8, and 6 apples compose the trio who first kept watch. Comparing our results with the expressions for each boy's share given above, we find,

First trio;—Jerry, Jeorje, and Josh; respective shares;—6, 13, 8. Second trio;—Jack, Jake, and Jim; respective shares;—5, 12, and 10.

$\therefore$ Jerry, Josh and Jeorje first kept watch.

Also solved by *A. L. FOOTE, H. C. WHITAKER*, and *G. B. M. ZERR.*

PROBLEMS.

21. Proposed by Professor J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

A tobacconist has two kinds of smoking tobacco, of which the price of the

better kind is $1 and of the inferior $.75 per ℔. Now, he takes 9 parts of the better and mixes it with 2 parts of the inferior, then 9 parts of the mixture with 2 parts of the inferior, etc. What is the price of the *nth* mixture per ℔.?

22. **Proposed by F. P. MATZ, M. A., M. Sc., Ph. D., Editor of the Department of Mathematics in the "New England Journal of Education" and Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.**

For the sum $D=\$30$, Messrs. Zerr and Ellwood contracted to plough the sod for a circular track, width $m=60$ feet and inner radius $r=940$ feet. How is the money to be divided, if they commence ploughing at the inner circumference of the track, make uniform furrows of width $n=1\frac{1}{3}$ feet, and Mr. Ellwood continually follows Mr. Zerr during the ploughing?

Solutions to these problems should be received on or before July 1st.

GEOMETRY.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

6. **Proposed by EARL D. WEST, West Middleburg, Logan County, Ohio.**

Having the sides 6, 4, 5, and 3 respectively of a trapezium, inscribed in a circle, to find the diameter of the circle.

II. **Solution by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia; and J. A. CALDERHEAD, Superintendent of Schools, Limaville, Ohio; and CHARLES E. MYERS, Canton, Ohio.**

Let $ABCD$ be the quadrilateral inscribed in a circle, AC the diagonal, $AB=a=5$, $BC=b=3$, $DC=c=6$, and DA$=d=4$.

From the triangles ABC and CDA

$$AC^2=a^2+b^2-2ab\cos B$$

$$AC^2=c^2+d^2-2cd\cos D=c^2+d^2+2cd\cos B$$

$$\therefore\ \cos B=\frac{a^2+b^2-c^2-d^2}{2(ab+cd)}.$$

$$\mathrm{Sin}^2 B=1-\frac{(a^2+b^2-c^2-d^2)^2}{4(ab+cd)^2}$$

$$=\frac{(c+b+d-a)(a+b+d-c)(a+c+d-b)(a+b+c-d)}{4(ab+cd)^2}$$

$$=\frac{16(s-a)(s-b)(s-c)(s-d)}{4(ab+cd)^2}\quad \text{where } s=\tfrac{1}{2}(a+b+c+d)$$

$$AC^2=c^2+d^2+\frac{2cd(a^2+b^2-c^2-d^2)}{2(ab+cd)}=\frac{(ac+bd)(ad+bc)}{(ab+cd)}.$$

$$R=\text{radius}=\frac{AC}{2\sin b}=\frac{1}{4}\sqrt{\frac{(ab+cd)(ac+bd)(ad+bc)}{(s-a)(s-b)(s-c)(s-d)}}$$

$$2R=D=\tfrac{1}{2}\sqrt{\frac{(39)(42)(38)}{(4)(6)(3)(5)}}=\tfrac{1}{2}\sqrt{172.9}=6.57457225.$$

Also solved by *P. S. BERG, H. C. WHITAKER, J. R. BALDWIN, P. H. PHILBRICK, J. W. SCHEFFER.*

[Note.—The formula for the area of an inscriptable quadrilateral is $A=\sqrt{s(s-a)(s-b)(s-b)(s-c)(s-d)}$, where $s=\frac{1}{2}(a+b+c+d)$.—Ed.]

7. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy in the Ohio University, Athens, Ohio.

Through each point of the straight line $x=my+h$ is drawn a chord of the parabola $y^2=4ax$, which is bisected in the point. Prove that this chord touches the parabola $(y+2am)^2=8a(x-h)$.

II. Solution by L. E. PRATT, Tecumseh, Nebraska.

Let $y=Bx+C\ldots.(1)$ be a straight line cutting the given parabola. The co-ordinates of the middle point of the chord intercepted by the curve are

$$\left(\frac{2a-BC}{B^2},\ \frac{2a}{B}\right).$$

Substituting these for x and y in the equations of the given straight line, we have

$$BC=2a-2amB-hB^2\ldots.(2).$$

If (x_1,y_1) be any point of (1) we have $y_1=Bx_1+C\ldots.(3)$.

Eliminating C from (2) and (3) we obtain a quadratic in B which may be written

$$B=\frac{y_1+2am\pm\sqrt{(y_1+2am)^2-8a(x_1-h)}}{2(x_1-h)}\quad\ldots.(4).$$

This result shows that two chords bisected by the straight line $x=my+h$ may be drawn through the point x_1,y_1; that when the roots of B, the angular coefficient, are equal the two chords coincide in one; that this takes place when the radical in (4) is equal to zero. But when the radical equals zero the point x_1,y_1, is a point of the parabola

$$(y+2am)^2=8a(x-h)$$

and the straight line (1) is tangent to it.

This problem was solved in a very excellent manner by *Professors HUME, SCHEFFER,* and *ZERR.*

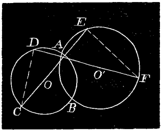
9. Proposed by J. C. GREGG, Superintendent of Schools, Brazil, Indiana.

Two circles intersect in A and B. Through A two lines CAE and DAF are drawn, each passing through a centre and terminated by the circumference. Show that $CA\times AE=DA\times AF$. [*Euclid.*]

Solution by Miss GRACE H. GRIDLEY, Student in Kidder Institute, Kidder, Missouri.

Let the straight lines CAE and DAF pass through the point of intersection A of the two circles, and through the centers O and O', respectively.

Connect by straight lines the points D and C, and E and F. Then the triangles CDA and AED are right triangles, being inscribed in a semi-circle. The two triangles are also similar, having the acute angle EAF of the one equal to the acute angle DAC of the other.



$\therefore CA : AF :: AD : AE$, the homologous sides of similar triangles being proportional.

$\therefore CA \times AE = AF \times AD$. Q. E. D.

Remark.—When the angle CAF is a right angle AE and AD each equal zero. When the angle CAF is less than a right angle, the point E will fall on the semi-circumference ABF and the point D will fall on the diameter AF and $CA \times AE = AF \times AD$ as before.

Also solved by *J. F. W. SCHEFFER, JOSIAH H. DRUMMOND, ROBERT J. ALEY, G. B. M. ZERR, J. A CALDERHEAD, P. S BERG.* and *P. H. PHILBRICK.*

10. Proposed by ERIC DOOLITTLE, Instructor in Mathematics, State University of Iowa, Iowa City.

If MN be any plane, and A and B any points without the plane, to find a point P, in the plane, such that $AP+PB$ shall be a minimum.

Solution by THADDEUS MERRIMAN, South Bethlehem, Pennsylvania.

First, suppose the points to be on opposite sides of the plane; the point where the straight line joining the given points pierces the plane is the required point P; since a straight line is the shortest distance between two points.

Second, suppose the points to be on the same side of the plane; let AB be the straight line joining them, CD the projection of AB on the plane MN, and AC the perpendicular which projects A on the plane. Now, produce AC to E making $AC=CE$, and join E and B; the point P where EB cuts CD is the required point. For, join A and P, and let Q be any other point in the plane; join A and Q, and B and Q, also Q and E. Now, since $AC=CE$ by construction, $AP=PE$ and $AQ=QE$; consequently $BQ+QE=BQ+QA$, and therefore, since $BE<BQ+QE$, we have $AP+PB<AQ+QB$, or $AP+PB$ is a minimum.

[This demonstration is by Thaddeus Merriman, the 17 year old son of Professor Mansfield Merriman.—ED.]

Also solved by *J. H. BEACH, G. B. M. ZERR, LEONARD E. DICKSON, F. A. SWANGER, H. C. WHITAKER, P. H. PHILBRICK, A. H. BELL* and *J. F. W. SCHEFFER.*

PROBLEMS.

31. Proposed by Professor G. I. Hopkins, Manchester, New Hampshire.

A field is bounded as follows: N. 14° W. 15.2 chains; N. 70°30′ E. 20.43 chains; S. 6° E. 22.79 chains; N. 86° 30′ W. 18 chains. A spring within it bears from the second corner S. 75° E. 7.9 chains. It is required to cut off 10 acres from the west side of the field by a straight fence through the spring. How far will it be from the first corner to the point at which the division fence meets the fourth side?

32. **Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics in the Ohio University, Athens, Ohio.**

If a conic be inscribed in a triangle and its focus move along a given straight line, the locus of the other focus is a conic circumscribing the triangle.

Solutions to these problems should be received on or before July 1st.

CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him,

SOLUTIONS TO PROBLEMS.

9. **Proposed by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.**

The solids bounded by the surfaces whose equations are $\left[\frac{x}{a}\right]^{\frac{2}{3}}+\left[\frac{y}{b}\right]^{\frac{2}{3}}+\left[\frac{z}{c}\right]^{\frac{2}{3}}=1$ and $x^{\frac{2}{3}}+y^{\frac{2}{3}}+z^{\frac{2}{3}}=b^{\frac{2}{3}}$ where $a>b>c$ have their centers coincident.

Find (1 and 2) the volume of each without the other, and (3) the volume common to both by direct integration, using the formula $V=\int\int\int dxdydz$.

Solution by the Proposer.

(1). From the equations we find for z,

$z=(b^{\frac{2}{3}}-x^{\frac{2}{3}}-y^{\frac{2}{3}})^{\frac{3}{2}}$ and $z=\frac{c}{ab}(a^{\frac{2}{3}}b^{\frac{2}{3}}-a^{\frac{2}{3}}y^{\frac{2}{3}}-b^{\frac{2}{3}}x^{\frac{2}{3}})^{\frac{3}{2}}$ eliminating z we find for the x—limits

$$x=\frac{a}{b}\left\{\frac{(b^{\frac{2}{3}}-c^{\frac{2}{3}})(b^{\frac{2}{3}}-y^{\frac{2}{3}})}{a^{\frac{2}{3}}-c^{\frac{2}{3}}}\right\}^{\frac{3}{2}}=x_1 \text{ to } x=0, \text{ the } y\text{—limits are } y=b \text{ to } y=0.$$

The volume of that part of $x^{\frac{2}{3}}+y^{\frac{2}{3}}+z^{\frac{2}{3}}=b^{\frac{2}{3}}$ without $\left(\frac{x}{a}\right)^{\frac{2}{3}}+\left(\frac{y}{b}\right)^{\frac{2}{3}}\left(\frac{z}{c}\right)^{\frac{2}{3}}=1$ we get

$$V=8\int_0^b\int_0^{x_1}(b^{\frac{2}{3}}-x^{\frac{2}{3}}-y^{\frac{2}{3}})^{\frac{3}{2}}dydx-\frac{8c}{ab}\int_0^b\int_0^{x_1}(a^{\frac{2}{3}}b^{\frac{2}{3}}-a^{\frac{2}{3}}y^{\frac{2}{3}}-b^{\frac{2}{3}}x^{\frac{2}{3}})^{\frac{3}{2}}\,dydx$$

$$V=8\int_0^b / \Big[\tfrac{1}{2}x(b^{\frac{2}{3}}-x^{\frac{2}{3}}-y^{\frac{2}{3}})^{\frac{3}{2}}+\frac{3}{8}x(b^{\frac{2}{3}}-y^{\frac{2}{3}})(b^{\frac{2}{3}}-x^{\frac{2}{3}}-y^{\frac{2}{3}})^{\frac{1}{2}}$$

$$-\frac{3}{16}x^{\frac{1}{3}}\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}\right)^{2}\left(b^{\frac{2}{3}}-x^{\frac{2}{3}}-y^{\frac{2}{3}}\right)^{\frac{1}{2}}+\frac{3}{16}$$

$$\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}\right)^{3}\sin^{-1}\left\{\frac{x}{(b^{\frac{2}{3}}-y^{\frac{2}{3}})^{\frac{3}{2}}}\right\}^{\frac{1}{3}}\Big]_{o}^{x_1}\Big/\,dy$$

$$-\frac{8c}{ab}\int_{o}^{b}\Big/\left[\frac{1}{2}x\left\{a^{\frac{2}{3}}\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}\right)-b^{\frac{2}{3}}x^{\frac{2}{3}}\right\}^{\frac{3}{2}}+\frac{3}{8}xa^{\frac{2}{3}}\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}\right)\right.$$

$$\left\{a^{\frac{2}{3}}\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}\right)-b^{\frac{2}{3}}x^{\frac{2}{3}}\right\}^{\frac{1}{2}}$$

$$-\frac{3}{16}\frac{x^{\frac{1}{3}}a^{\frac{4}{3}}(b^{\frac{2}{3}}-y^{\frac{2}{3}})^{2}}{b^{\frac{2}{3}}}\left\{a^{\frac{2}{3}}\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}\right)-b^{\frac{2}{3}}x^{\frac{2}{3}}\right\}^{\frac{1}{2}}$$

$$+\frac{3a^{2}}{16b}\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}\right)^{3}\sin^{-1}\left\{\frac{bx}{a(b^{\frac{2}{3}}-y^{\frac{2}{3}})^{\frac{3}{2}}}\right\}^{\frac{1}{3}}\Big]_{o}^{x_1}\Big/\,dy$$

$$V=\left[\frac{3}{2}\sin^{-1}\left\{\frac{a}{b}\frac{(b^{\frac{2}{3}}-c^{\frac{2}{3}})}{(a^{\frac{2}{3}}-c^{\frac{2}{3}})}\right\}^{\frac{1}{3}}-\frac{3ac}{2b^{2}}\sin^{-1}\left\{\frac{b^{\frac{2}{3}}-c^{\frac{2}{3}}}{a^{\frac{2}{3}}-c^{\frac{2}{3}}}\right\}^{\frac{1}{3}}\right.$$

$$+\frac{3ac}{b^{2}}\left\{\frac{(b^{\frac{2}{3}}-c^{\frac{2}{3}})^{\frac{5}{2}}(a^{\frac{2}{3}}-b^{\frac{2}{3}})^{\frac{1}{2}}}{c^{\frac{2}{3}}(a^{\frac{2}{3}}-c^{\frac{2}{3}})^{2}}\right\}$$

$$\left.+\frac{3ac}{2b^{2}}\left\{\frac{a^{\frac{2}{3}}c^{\frac{2}{3}}-b^{\frac{4}{3}}}{a^{\frac{2}{3}}c^{\frac{2}{3}}}\right\}\left\{\frac{(b^{\frac{2}{3}}-c^{\frac{2}{3}})^{\frac{1}{2}}(a^{\frac{2}{3}}-b^{\frac{2}{3}})^{\frac{1}{2}}}{a^{\frac{2}{3}}-c^{\frac{2}{3}}}\right\}\right]\int_{o}^{b}\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}\right)^{3}dy.$$

$$V=\frac{8}{35}b^{3}\sin^{-1}\left\{\frac{a}{b}\frac{(b^{\frac{2}{3}}-c^{\frac{2}{3}})}{(a^{\frac{2}{3}}-c^{\frac{2}{3}})}\right\}^{\frac{1}{3}}-\frac{8}{35}abc\sin^{-1}\left\{\frac{b^{\frac{2}{3}}-c^{\frac{2}{3}}}{a^{\frac{2}{3}}-c^{\frac{2}{3}}}\right\}^{\frac{1}{3}}$$

$$+\frac{16}{35}abc\left\{\frac{(b^{\frac{2}{3}}-c^{\frac{2}{3}})^{\frac{5}{2}}(a^{\frac{2}{3}}-b^{\frac{2}{3}})^{\frac{1}{2}}}{c^{\frac{2}{3}}(a^{\frac{2}{3}}-c^{\frac{2}{3}})^{2}}\right\}$$

$$+\frac{8}{35}abc\left\{\frac{a^{\frac{2}{3}}c^{\frac{2}{3}}-b^{\frac{4}{3}}}{a^{\frac{2}{3}}c^{\frac{2}{3}}}\right\}\left\{\frac{(b^{\frac{2}{3}}-c^{\frac{2}{3}})^{\frac{1}{2}}(a^{\frac{2}{3}}-b^{\frac{2}{3}})^{\frac{1}{2}}}{a^{\frac{2}{3}}-c^{\frac{2}{3}}}\right\}$$

(2) From the equations $x=\frac{a}{bc}\left\{c^{\frac{2}{3}}\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}\right)-b^{\frac{2}{3}}z^{\frac{2}{3}}\right\}^{\frac{3}{2}}$, and $x=\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}-z^{\frac{2}{3}}\right)^{\frac{3}{2}}$

$\therefore$ z-limits are $z=\frac{c}{b}\left\{\frac{(b^{\frac{2}{3}}-a^{\frac{2}{3}})(b^{\frac{2}{3}}-y^{\frac{2}{3}})}{c^{\frac{2}{3}}-a^{\frac{2}{3}}}\right\}^{\frac{3}{2}}$ to $z=0$. y-limits$=b$ and 0.

Hence the volume of that part of $\left(\frac{x}{a}\right)^{\frac{2}{3}}+\left(\frac{y}{b}\right)^{\frac{2}{3}}+\left(\frac{z}{c}\right)^{\frac{2}{3}}=1$, without

$x^{\frac{2}{3}}+y^{\frac{2}{3}}+z^{\frac{2}{3}}=b^{\frac{2}{3}}$ we get

$$V=8\frac{a}{bc}\int_0^b\int_0^{z_1}\left\{c^{\frac{2}{3}}(b^{\frac{2}{3}}-y^{\frac{2}{3}})-b^{\frac{2}{3}}z^{\frac{2}{3}}\right\}^{\frac{3}{2}}dydz-8\int_0^b\int_0^{z_1}\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}-z^{\frac{2}{3}}\right)^{\frac{3}{2}}dydz.$$

By writing a for c and c for a and $+$ for $-$ this integral is the same as the first, hence by symmetry.

$$V=\frac{8}{35}abc\sin^{-1}\left\{\frac{b^{\frac{2}{3}}-a^{\frac{2}{3}}}{c^{\frac{2}{3}}-a^{\frac{2}{3}}}\right\}^{\frac{1}{3}}-\frac{8}{35}b^3\sin^{-1}\left\{\frac{c\,(b^{\frac{2}{3}}-a^{\frac{2}{3}})}{b\,(c^{\frac{2}{3}}-a^{\frac{2}{3}})}\right\}^{\frac{1}{3}}$$

$$-\frac{16}{35}abc\left\{\frac{(b^{\frac{2}{3}}-a^{\frac{2}{3}})^{\frac{5}{2}}(c^{\frac{2}{3}}-b^{\frac{2}{3}})^{\frac{1}{2}}}{a^{\frac{2}{3}}(c^{\frac{2}{3}}-a^{\frac{2}{3}})^2}\right\}$$

$$-\frac{8}{35}abc\left\{\frac{a^{\frac{2}{3}}c^{\frac{2}{3}}-b^{\frac{4}{3}}}{a^{\frac{2}{3}}c^{\frac{2}{3}}}\right\}\left\{\frac{(b^{\frac{2}{3}}-a^{\frac{2}{3}})^{\frac{1}{2}}(c^{\frac{2}{3}}-b^{\frac{2}{3}})^{\frac{1}{2}}}{c^{\frac{2}{3}}-a^{\frac{2}{3}}}\right\}.$$

(3). For volume common to both x-limits are $x=x_1$ to $x=0$ and $x=\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}\right)^{\frac{3}{2}}=x_2$ to $x=x_1$, y-limits are $y=b$ to $y=0$.

$$\therefore\ V=8\int_0^b\int_0^{x_1}\frac{c}{ab}\left\{a^{\frac{2}{3}}(b^{\frac{2}{3}}-y^{\frac{2}{3}})-b^{\frac{2}{3}}x^{\frac{2}{3}}\right\}^{\frac{3}{2}}dydx$$

$$+8\int_0^b\int_{x_1}^{x_2}\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}-x^{\frac{2}{3}}\right)^{\frac{3}{2}}dydx.$$

$$V=\left[\frac{3}{4}\pi+\frac{3}{2}\frac{ac}{b^2}\sin^{-1}\left\{\frac{b^{\frac{2}{3}}-c^{\frac{2}{3}}}{a^{\frac{2}{3}}-c^{\frac{2}{3}}}\right\}^{\frac{1}{3}}-\frac{3}{2}\sin^{-1}\left\{\frac{a\,(b^{\frac{2}{3}}-c^{\frac{2}{3}})}{b\,(a^{\frac{2}{3}}-c^{\frac{2}{3}})}\right\}^{\frac{1}{3}}\right.$$

$$-\frac{3ac}{b^2}\left\{\frac{(b^{\frac{2}{3}}-c^{\frac{2}{3}})^{\frac{5}{2}}(a^{\frac{2}{3}}-b^{\frac{2}{3}})^{\frac{1}{2}}}{c^{\frac{2}{3}}(a^{\frac{2}{3}}-c^{\frac{2}{3}})^2}\right\}-\frac{3ac}{2b^2}\left\{\frac{a^{\frac{2}{3}}c^{\frac{2}{3}}-b^{\frac{4}{3}}}{a^{\frac{2}{3}}c^{\frac{2}{3}}}\right\}$$

$$\times\left\{\frac{(b^{\frac{2}{3}}-c^{\frac{2}{3}})^{\frac{1}{2}}(a^{\frac{2}{3}}-b^{\frac{2}{3}})^{\frac{1}{2}}}{a^{\frac{2}{3}}-c^{\frac{2}{3}}}\right\}\Bigg]\int_0^b\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}\right)^3dy.$$

$$\therefore\ V=\frac{4}{35}\pi b^3+\frac{8}{35}\ abc\ \sin^{-1}\left\{\frac{b^{\frac{2}{3}}-c^{\frac{2}{3}}}{a^{\frac{2}{3}}-c^{\frac{2}{3}}}\right\}^{\frac{1}{3}}-\frac{8}{35}\ b^3\sin^{-1}\left\{\frac{a(b^{\frac{2}{3}}-c^{\frac{2}{3}})}{b\ (a^{\frac{2}{3}}-c^{\frac{2}{3}})}\right\}^{\frac{1}{3}}$$

$$-\frac{16}{35}\ abc\left\{\frac{(b^{\frac{2}{3}}-c^{\frac{2}{3}})^{\frac{5}{2}}(a^{\frac{2}{3}}-b^{\frac{2}{3}})^{\frac{1}{2}}}{a^{\frac{2}{3}}(c^{\frac{2}{3}}-a^{\frac{2}{3}})^2}\right\}$$

$$-\frac{8}{35}\ abc\left\{\frac{a^{\frac{2}{3}}c^{\frac{2}{3}}-b^{\frac{4}{3}}}{a^{\frac{2}{3}}c^{3}}\right\}\left\{\frac{(b^{\frac{2}{3}}-c^{\frac{2}{3}})^{\frac{1}{2}}(a^{\frac{2}{3}}-b^{\frac{2}{3}})^{\frac{1}{2}}}{a^{\frac{2}{3}}-c^{\frac{2}{3}}}\right\}.$$

Cor. If $b=c$, $V=\frac{4}{35}\pi b^3$.

If $a=b$, $V=\frac{4}{35}\pi a^2c$.

Also solved by *M. C. STEVENS*.

PROBLEMS.

18. Proposed by J. M. BANDY, Professor of Mathematics, Elon College, North Carolina.

If the ordinate ST of any point T on a circle

$$x^2+y^2=r^2$$

be produced so that $ST.TP=r^2$, prove that the whole area between the locus of P and its asymptotes is double the area of the circle.

19. Proposed by A. L. FOOTE, No. 830, Broad Street, New York City.

A and B are in a circular room 30 feet in diameter, A being at the center and B at the circumference. B runs around at the rate of 600 feet per minute and A persues him at the rate of 100 feet per minute. How long will the race last, and how far will each have traveled till B is caught.

Solutions to these problems should be received on or before July 1st.

MECHANICS.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

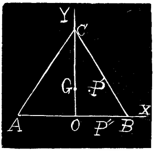
SOLUTIONS TO PROBLEMS.

5. Proposed by J. R. BALDWIN, A. M., Professor of Mathematics and Commercial Law, Davenport Business College, Davenport, Iowa.

A 200 pound ball lies on a three legged table, having the legs equally distant apart and perpendicular to the plane of the top of the table. (1) What is the weight on each leg of the table not including the top when the ball is 2 feet, 3 feet, and 4 feet distant from the three legs? (2) If the ball is 2 feet, 3 feet, and 5 feet from the legs, what must be the weight of the top to keep from tipping and the weight on each leg excluding the top and and also including the top?

Solution by ALFRED HUME, C. E., D. Sc., Professor of Mathematics in the University of Mississippi.

Let P' be the first position of ball, and A, B, and C the three legs of table.



Let these four letters, also, represent the weights at the points designated. Refer these points to the rectangular axes, OX and OY, O being the middle point of AB, and let $AB=2a$. The points A, B, C, P', will be respectively $(-a,\ 0),(a,\ 0),(0,\ \surd 3\ a),(x,\ y)$. The distances of P' from A, B, and C being 4, 2, and 3 respectively, $(x+a)^2+y^2=16$, $(x-a)^2+y^2=4$, $x^2+(y-\surd 3\ a)^2=9$. From these equations, $a=2.4749$, $x=1.2122$, $y=1.5484$. (None of these values are exact, nor are any that follow).

Taking moments about OX, weight at $C=72+$.

Taking moments about OY,

$$200x=B.a-A.a=(B-A)\ a=(128-2A)a$$

and therefore, pressure at $A=15+$, and that at $B=113-$.

Let the second position of the ball be P'', the distances to A, B, and C being respectively, 3, 2, and 5. By process similar to the above the new coordinates of P are found to be, (0.5051,0.346).

If G is the center of gravity of the table and if G denotes the weight of the table then, in order that turning may not occur,

$$G\times\frac{\surd 3\ a}{3}=P\times 0.346, \text{ and } G=48+.$$

Neglecting the weight of the table a force of 16 pounds must be applied downwards at C to balance P.

Then the sum of pressures at A and $B=216$, and, taking moments about OY, $(B-A)a=200x$,

$$(216-2A)a=200x,$$
$$A=88,\ B=128.$$

When weight of table is considered $A=104$, $B=144$, $C=0$.
Summing up these results,

with weight at P'................$A=15$, $B=113$, $C=72$.
" " " P'' (neglecting G) $A=88$, $B=128$, $C=-16$.
" " " P'' (considering G) $A=104$, $B=144$, $C=0$.

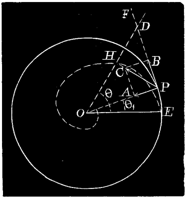
4. Proposed by DeVOLSON WOOD, M. A., C. E., Professor of Mechanical Engineering, Stevens Institute of Technology, Hoboken, New Jersey.

A particle starts at rest and revolves in a circle with a uniform acceleration, acquiring a velocity v in t seconds. Find the locus of the foot of the perpendicular from the centre of the circle upon the resultant acceleration.

A graphical solution by ALFRED HUME, C. E.,D. Sc., Professor of Mathematics in the University of Mississippi.

CONSTRUCTION OF LOCUS.

Let the particle start from rest at E arriving at any point P in time, t, with velocity v. Let PB, PA, and PC be respectively the tangential, radial, and resultant, accelerations.



Then $PB=\frac{v}{t}$, $PA=\frac{v^2}{R}$ (numerically), and, since the acceleration in the circular path is uniform, arc $PE=\frac{1}{2}\,vt$.

Now, drawing OH perpendicular to PC and denoting angles HOE and POE by θ and θ_1 respectively, arc $PE=R\theta_1=\frac{1}{2}\,vt$, and

$$\tan(\theta-\theta_1)=\tan CPB=\frac{BC}{BP}=\frac{v^2}{R}\cdot\frac{t}{v}=\frac{vt}{R}$$
$$=\frac{2R\theta_1}{R}=2\theta_1.$$

Therefore $PD=2$ arc PE, D being the intersection of PB and OH, both produced.

Hence the construction:—To find the point on required locus corresponding to P, any position of the particle, lay off on the tangent PF a distance PD equal to twice the arc PE; connect D with the center of the circle O; from P drop a perpendicular to OD meeting OD at H. H is the point required.

II. Solution by E. C. MURPHY, C. E., University of Kansas, Lawrence, Kansas.

Let Fig. 1 represent a circle of radius R on the circumference of which a particle is moving with a unform acceleration p, having started from rest at E. Let P be the position of the particle after time t when its velocity is V and its normal acceleration is $\frac{V^2}{R}$.

Let ρ be the perpendicular dropped from the center of the circle on the resultant acceleration PC; α=the angle OPC and PA,the normal acceleration.

Then from the Fig. we have

$$\rho=R\sin\alpha\ldots.(1).$$

From the triangle APC we have

$$\cos\alpha=\frac{AP}{PC}=\frac{V^2}{pR}\text{ or }\alpha=\cos^{-1}\left(\frac{V^2}{pR}\right)\ldots.(2).$$

$$p=\frac{dV}{dt}\text{ or }dV=pdt.$$

$$\int_0^V dV=p\int_0^t dt\text{ or }V=pt\ldots.(3).$$

Substituting for V its value in (2) and for α its value in (1) we have

$$\rho=R\sin\left[\cos^{-1}\left(\frac{pt^2}{R}\right)\right]\ldots.(4).$$

$\rho=0$ and the direction of the resultant acceleration is through the center of circle when

$$\cos^{-1}\left(\frac{pt^2}{R}\right)=1\text{ or when }t=\sqrt{\frac{R}{p}}.$$

If it is the tangential acceleration which is constant and$=p$, then

$$\rho=R\sin\left[\tan^{-1}\left(\frac{pt^2}{R}\right)\right]\ldots.(5).$$

PROBLEMS.

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11. Proposed by CHARLES E. MYERS, Canton, Ohio.

"A homogeneous sphere moves down a rough inclined plane, whose angle of inclination θ to the horizon is greater than that of the angle of friction; if the coefficient of friction is less than $\frac{2}{7}$ tang. θ, show that the sphere will roll and slide down the inclined plane."

12. Proposed by J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

A horizontal plane without weight is supported on three points A, B, C. A weight W is laid upon the table at a point G. If $AG=a$, $BG=b$, $CG=c$, $\angle AGB=\beta$, $\angle AGC,=\gamma$; find the pressures upon A, B, C.

Solutions to these problems should be received on or before July 1st.

* * *

DIOPHANTINE ANALYSIS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTION TO THE CELEBRATED INDETERMINATE EQUATION.

$$x^2-Ny^2=\pm 1.$$

By A. H. BELL, Hillsboro, Illinois.

[Continued from the March Number.]

No. 5. Example: Given $x^2-94y^2=1$. Required x and y.

This is the *most difficult number* under a 100. Proceeding in the usual way we have,

No. Term	1	2	3	4	5	6
			+3			
$m=$	2 +	1	4	7	10	13
			+29			
$n=$	10 +	10	39	68	97	126
1st series of Diff.	+15	−6	−17	−18	−9	+10
D_1.	−21	−11	−1	+9	+19	
D_2.	+10....					

Preliminary for extending the series, because −9 and +10 are nearly of a size.

$3+10^{+13}\ 23$
$29+97^{+126}\ 223$
$+5-9-3$
D_1-14+6
D_2+20

We proceed with the two last convergents and commence a new series.

2nd series No. Terms			3	4	5	6	7	8
		+23						
$m=$	19 +	13	36	59	82	105	128	151
		+223						
$n=$	97	+126	349	572	795	1018	1241	1464
Diff.	−9	+10	+23	+30	+31	+26	+15	−2
D_1	19,	+ 13,	+7	+1	−5	−11	−17	

giving $y=221064$.
$y^2=48869292096$

$D_2=-\!-6$ .... $x=\dfrac{151\times94\times151\times1464}{1464}+1$.

$x=2.143.195.$

Proof $\begin{cases} 2143295^2 \quad\quad 94\times221064^2 \quad x^2=4583713457025 \\ 4593713457025-4593713457024=1 \end{cases}$

These are the least values that can be found for x and y.

(TO BE CONTINUED.)

SOLUTIONS TO PROBLEMS.

3. Proposed by O. S. KIBLER, Superintendent of Schools, West Middleburg, Logan County, Ohio.

It is required to find three whole numbers in an arithmetical progression, such that the sum of every two of them shall be a square.

III Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Represent the required numbers by $\frac{1}{2}x^2-(2x+1)$, $\frac{1}{2}x^2$, and $\frac{1}{2}x^2+$

$(2x+1)$; then must $x^2-(2x+1)$, x^2, and $x^2+(2x+1)$ be squares. Now, all we have to do is to find a value of x that will render $x^2-(2x+1)$ a square.

Put $x^2-(2x+1)=(x-p)^2$; then

$$x=\tfrac{1}{2}(p^2+1)\div(p-1).$$

Put $p=2$, then numbers are:—46, +50, +146;

" $p=8$, " " " : 386, 8450, 16514;

" $p=9$, " " " : 482, 3362, 6242;

" $p=10$, " " " : 4562, 20402, 36242;

" $p=11$, " " " : 2162, 7442, 12722.

&c. &c. &c.

PROBLEMS.

11. **Proposed by ARTEMAS MARTIN, LL. D., U. S. Coast Survey Office, Washington, D. C.**

Find three whole numbers such that the square of the sum of any two of them diminished by the square of the other number shall be a square.

12. **Proposed by H. W. DRAUGHON, Clinton, Louisiana.**

Find three numbers such that, the sum of their cubes may be a square, and the sum of their squares a cube.

Solutions to these problems should be received on or before July 1st.

AVERAGE AND PROBABILITY.

onducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

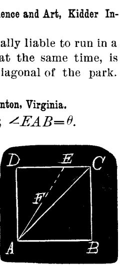
SOLUTIONS TO PROBLEMS.

3. **Proposed by MISS LECTA MILLER, B. L., Professor of Natural Science and Art, Kidder Institute, Kidder, Missouri.**

A deer, wounded at the corner of a square park, is equally liable to run in a straight line in any direction, from the corner of the park, and, at the same time, is also liable to drop dead before running a distance equal to the diagonal of the park. What is the chance that the deer will drop dead in the park?

I. **Solution by Professor G. B. M. ZERR, Principal of High School, Staunton, Virginia.**

Let $ABCD$ be the square park, side unity; $AF=r$; $\angle EAB=\theta$.



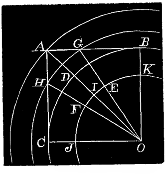
Then $AC=\sqrt{2}$, $AE=\text{cosec}\theta$.

$$\therefore \text{ chance }=p=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_0^{\text{cosec}\theta} rd\theta dr \Big/ \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}\int_0^{\sqrt{2}} rd\theta dr.$$

$$p=\frac{1}{\pi}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_0^{\text{cosec}\theta} rd\theta dr=\frac{1}{2\pi}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\text{cosec}^2\ \theta d\theta=\frac{1}{2\pi}.$$

II. **Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.**

Let $OB=OC=OD=s$, $OI=x$, $OG=OH=w$; then $OA=s\sqrt{2}$, area of $\square\, ABOC=s^2$, and area of circle radius of which is $OA=2\pi s^2$.



First Solution.—Knowing the *favorable* chances and the *whole number* of chances, the required chance becomes

$$C=\frac{s^2}{2\pi s^2}=\frac{1}{2\pi}\cdots\cdots\cdots(A).$$

SECOND SOLUTION.

The arc JIK increases for all values of OI less than OD; and the arc HG decreases for all values of OG greater than OD. The required chance, therefore, becomes

$$C=\frac{1}{2\pi s^2}\left\{\frac{\pi}{2}\int_0^s xdx+\int_s^{s\sqrt{2}}\left[\frac{\pi}{2}-2\cos^{-1}\left(\frac{s}{w}\right)\right]wdw\right\}$$

$$=\frac{1}{8}+\frac{1}{2\pi^2}\left[\frac{\pi s^2}{4}-2\int_s^{s\sqrt{2}}\cos^{-1}\left(\frac{s}{w}\right)wdw\right]$$

$$=\frac{1}{4}-\frac{1}{2\pi s^2}\left[w^2\cos^{-1}\left(\frac{s}{w}\right)-s\sqrt{(w^2-s^2)}\right]_s^{s\sqrt{2}}$$

$$=\frac{1}{4}-\frac{1}{2\pi}\left[\frac{\pi}{2}-1\right]=\frac{1}{2\pi}\cdots\cdots\cdots(B).$$

This problem was solved with different results by *H. W. DRAUGHON*, *P. H. PHILBRICK*, and *WILLIAM B. MILWARD*. If we have space, some of these solutions will be published next month.

PROBLEMS.

6. **Proposed by Professor J. F. W. SCHEFFER, Hagerstown, Maryland.**

Find the average length of all the diameters that can be drawn in a given ellipse.

7. **Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.**

A letter received is known to have come either from *Oshkosh* or *Ashland*. The only two consecutive letters legible on the postmark are SH. What is the probability that the letter received came from Oshkosh?

8. **Proposed by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.**

Prove that the mean area of all triangles having their vertices upon the surface of a given triangle and bases parallel to the base of the given triangle, is $\frac{13}{210}$ (area of given triangle).

Solutions to these problems should be received on or before July 1st.

MISCELLANEOUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

4. Proposed by J. K. ELLWOOD, A. M., Principal of Colfax School, Pittsburg, Pennsylvania

I have two circular grindstones, each $\frac{1}{2}$ in. thick. One is 6 in. and the other $4\frac{1}{2}$ in. in diameter, the aperture at center of each being $1\frac{1}{2}$ in. If when in motion they are continually tangent to each other, and $\frac{1}{2}$ cu. in. is ground off the larger wheel and $\frac{1}{4}$ cu. in. off the smaller in the first hour, how must their speed be increased so that the same amount per hour may be ground off each wheel until one is worn out? If in the first hour the larger wheel makes a revolutions, and the smaller b, how many must each make in each succeeding hour?

I. Solution by ALFRED HUME, C. E., Professor of Mathematics, University of Mississppi, University P. O., Mississippi.

It is assumed that the stones have independent motions, and that the diminution of their diameters caused by the mutual sliding friction is directly proportional to the number of revolutions. Let $ABCD$ be a rectangle which, revolving about CD, generates the larger grindstone. Let CE be the stone's radius at the end of the first hour. The volume ground off is, by the Theorem of Pappus,

$$2\pi\left(CA-\frac{AE}{2}\right)(AB\times AE);$$ or, substituting the given values, $CA=3$, $AB=\frac{1}{2}$, and the volume being $\frac{1}{2}$ cu in., AE is found to be $3-\sqrt{9-\frac{1}{\pi}}$. At the end of the first hour, then, the radius of the stone, CE, is $\sqrt{9-\frac{1}{\pi}}$. In the same way it may be shown that at the end of the second hour, another $\frac{1}{2}$ cu. in. having been ground off, the radius is $\sqrt{9-\frac{2}{\pi}}$.

At the end of the third hour, the radius is $\sqrt{9-\frac{3}{\pi}}$.

At the end of the nth hour, the radius is $\sqrt{9-\frac{n}{\pi}}$.

If the stone makes a revolutions during the first hour, during the second it must make $\dfrac{\sqrt{9-\frac{1}{\pi}}-\sqrt{9-\frac{2}{\pi}}}{3-\sqrt{9-\frac{1}{\pi}}}a$, and during the third,

$$\frac{\sqrt{9-\frac{2}{\pi}}-\sqrt{9-\frac{3}{\pi}}}{3-\sqrt{9-\frac{1}{\pi}}}a.$$

The revolutions made in the second, 3rd, and nth hours are, therefore,

$$(\sqrt{9\pi}+\sqrt{9\pi-1})(\sqrt{9\pi-1}-\sqrt{9\pi-2})\,a,$$
$$(\sqrt{9\pi}+\sqrt{9\pi-1})(\sqrt{9\pi-2}-\sqrt{9\pi-3})\,a,$$
$$(\sqrt{9\pi}+\sqrt{9\pi-1})(\sqrt{9\pi-n+1}-\sqrt{9\pi-n})\,a, \text{ respectively.}$$

The time required to grind away all of the stone is given by the equation,

$\sqrt{9-\frac{n}{\pi}}=\frac{3}{4}$, from which $n=\frac{135\pi}{16}$, the number of hours.

Similarly, the number of revolutions made by the smaller stone during the nth hour is found to be $\frac{1}{8}(\sqrt{81\pi}+\sqrt{81\pi-8})(\sqrt{81\pi-8(n-1)}-\sqrt{81\pi-8n})$ b, and the time required to grind it away, $\frac{69\pi}{8}$. Therefore the larger wears out first, and at this time the smaller is a cylindrical shell whose thickness is $\frac{3}{4}(\sqrt{\frac{3}{2}}-1)$ inches.

This problem was also solved by *P. H. PHILBRICK*, and *H. W. DRAUGHON*.

5. Proposed by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

A cubic mile of saturated air at 18°C. is cooled to a temperature of 10°C. How many tons of rain will fall?

Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

According to Silliman's Table the weight of the aqueous vapor in a cubic foot of saturated air at 18°C.,$=64\frac{2}{5}$°F.,$=6.663$ avoirdupois grains; and the weight of that in a cubic foot of the same kind of air at 10°C., $=50$°F., $=4.089$ avoirdupois grains. The difference of these weights is the weight of the rain that will fall from a cubic foot of air. Hence the weight of the rain that will fall from a cubic mile of air is

$$R=\frac{(5280)^3\times1287}{7000\times2000\times500}=27125.776 \text{ tons.}$$

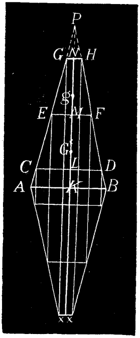
Also solved by the *PROPOSER*.

6. Proposed by H. C. WHITAKER, B. S., M. E., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.

Two men wish to buy a grindstone 42 inches in diameter and one foot thick at the center. To what thickness at the outer edge should the stone uniformly taper from the center so that each man may grind off 18 inches of the diameter and both have equal shares, the central six inches of the diameter being waste?

I. Solution by Professor P. H. PHILBRICK, M. S., C. E, Lake Charles, Louisiana; and A. L. FOOTE, No. 80 Broad Street, New York City.

Let $ABHG$ represent a half section of the stone through the centre. Draw the centre line $KLMN$; make $KL=3$, $LM=MN=9$ inches; and draw CD and EF parallel to AB. Let $GH=x$, and let G and g be the centers of gravity of $CDEF$ and $EFGH$. It is easy to show that,



$$EF=x+\frac{9}{21}(12-x)=\frac{4x+36}{7}\ldots.(1),$$

$$\text{and } CD=x+\frac{18}{21}(12-x)=\frac{x+72}{7}\ldots.(2).$$

$$\text{From Mechanics, } GL=\frac{1}{3}LM.\frac{CD+2EF}{CD+EF}=3.\frac{9x+144}{5x+108}$$

$$\text{and, } gM=\frac{1}{3}MN.\frac{EF+2HG}{EF+HG}=3.\frac{18x+36}{11x+36}.$$

$$\therefore\ GK=GL+3=6.\frac{7x+126}{5x+108}\ldots.(3),$$

$$\text{and } gK=gM+12=6.\frac{31x+90}{11x+36}\ldots.(4).$$

Let a=area of $EFGH$, A=area of $CDEF$, and V=volume ground off by each man.

$$\text{Now, } A=\tfrac{1}{2}LM(CD+EF)=\frac{9}{14}(5x+108)\ldots.(5),$$

$$\text{and } a=\tfrac{1}{2}MN(EF+GH)=\frac{9}{14}(11x+36)\ldots.(6).$$

Now, Vol. ground off by the first man = area $EFGH$ multiplied by the circumference of the circle described by radius Kg in revolving about AB as an axis. Hence, $V=2\pi(gK)a\ldots.(7)$.

Similarly, Vol. ground off by the second man = $V=2\pi(GK)A\ldots.(8)$. Substituting in these equations, omitting common factors, and equating we have $31x+90=7x+126$. $\therefore\ x=\frac{3}{2}=1\frac{1}{2}$ inches.

II. **Solution by SETH PRATT, C. E., Assyria, Michigan.**

1st. Let $2x$= the thickness of the stone at the outer edge. The stone is composed of one cylinder and two cones. R=21 inches =the radius of the cylinder and of each cone. Height of cylinder $=2x$. Height of each cone $=6-x$.

$$\text{Content of cylinder }=2R^2n'x=+882n'x.$$

$$\text{Content of the two cones }=R^2n'\tfrac{2}{3}(6-x)=1764n'-294n'x.$$

$$\text{Sum = content of stone }=1764n'+588n'x\ldots.(1).$$

2nd. R=12 inches = radius of cylinder and cones.

$$21:12::6-x:h=\frac{24-4x}{7}=\text{height of each cone.}$$

$$12-2h=\frac{36+8x}{7}=\text{height of cylinder.}$$

$$\text{Content of cylinder}=R^2n'.\left(\frac{36+8x}{7}\right)=\frac{5184n'}{7}+\frac{1152n'x}{7}.$$

$$\text{Content of cones}=R^2n'.\tfrac{2}{3}\left(\frac{24-4x}{7}\right)=\frac{2304n'}{7}-\frac{384n'x}{7}.$$

$$\text{Sum}=\frac{7488n'}{7}+\frac{768n'x}{7}\ldots.(2).$$

3rd. $R=3$ inches = radius of cylinder and cones.

$$21 : 3 :: 6-x : h'=\frac{6-x}{7}=\text{height of each cone.}$$

$$12-2h'=\frac{72+2x}{7}=\text{height of cylinder.}$$

$$\text{Content of cylinder}=R^2n'\left(\frac{72+2x}{7}\right)=\frac{648n'}{7}+\frac{18n'x}{7}.$$

$$\text{Content of cones}=R^2n'.\ \tfrac{2}{3}\left(\frac{6-x}{7}\right)=\frac{36n'}{7}-\frac{6n'x}{7}.$$

$$\text{Sum}=\frac{684n'}{7}+\frac{12n'x}{7}\ldots\ldots(3).$$

$$\text{From (1) take (2)}=\frac{4860n'}{7}+\frac{3348n'x}{7}\ldots\ldots(4).$$

$$\text{From (2) take (3)}=\frac{6804n'}{7}+\frac{756n'x}{7}\ldots\ldots(5).$$

Equate (4) and (5) and reduce, and $x=\frac{3}{4}$, or $2x=1\frac{1}{2}$ inches.

Also solved by *H. W. DRAUGHON, H. C. WHITAKER, ALFRED HUME, C. E. MYERS, G. B. M. ZERR*, and *W. L. HARVEY*.

PROBLEMS.

10. Proposed by SAMUEL HART WRIGHT, M. D., M. A., Ph. D., Penn Yan, Yates Co., N. Y.

A small cloud in the S. E. and altitude 70°, was soon after N. 60° E. with an altitude of 30°. In what direction was the wind blowing, the track of the cloud being the arc of a great circle?

11. Proposed by CHAS. E. MYERS, Canton, Ohio.

"Assuming the earth's orbit to be a circle, if a comet move in a parabola around the sun and in the plane of the earth's orbit, show that the comet cannot remain within the earth's orbit longer than 78 days."

12. Proposed by F. P. MATZ, M. S., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

If the measures of curvature and tortuosity of a curve be constant at every point of a curve, the curve will be a helix traced on a cylinder.

QUERIES AND INFORMATION.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

Answer to Queries in the American Mathematical Monthly for March 1894. (Vol. I. No. 3. page 102.)

I. Omitting Euclid's Parallel-Postulate, but taking for granted all his other postulates and "common notions", it follows by Eu. I. 27, that two coplanar

straights perpendicular to a third are parallel; that is "being produced ever so far both ways *do* not meet".

II. Euclid never admits any hypothetical construction beyond his actual postulates, so that Eu. I. 27 is in harmony both with von Staudt's extention of Euclidean space, in accordance with which two coplanar straights perpendicular to a third have at infinity a common point; and also in harmony with Lobatschewsky's non-Euclidean space, in which two coplanar straights perpendicular to a third have no point in common, not even at infinity.

III. The assumption, for spherics and single elliptic geometry, that two straights perpendicular to a third do intersect without going to infinity, is deliberately taken as the contradictory of Labatschewsky's theorem 4.

GEORGE BRUCE HALSTED, TRANSLATOR OF LOBATSCHEWSKY.

L. B. Hayward's solution of Ex. 3, p. 19, Jan. No., is not correct. Mr. Hayward says the loss is $3\%+2\frac{91}{100}\%$ or $5\frac{91}{100}\%$ while it is only $2\frac{91}{100}\%$. If, in the problem, he had bought it back for 97% of what he paid for it originally, that is, for exactly what he sold it for, he would have paid back exactly what he received for it and of course would have lost nothing. He loses then only the amount he pays over what he sold it for that is $2\frac{91}{100}\%$.

Or look at it another way. It is assumed that the stock is worth par. Then if he sells for 97% he loses 3%. Then if he buys back at $99\frac{91}{100}\%$ he is a gainer by $\frac{9}{100}\%$ as it is worth par. That is, again he loses $(3\%-\frac{9}{100}\%)=2\frac{91}{100}\%$.

The cost, therefore, is $\$12\div0.02\frac{91}{100}$; or $412.37+

W. F. BRADBURY.

The criticisms made by Prof. Black (No. 3, p. 67) is now inapplicable as the demonstration given in Wentworth's Plane & Solid Geometry, Revised Edition 1891, is made valid by the first sentence in the proof: "If the two pyramids are not equivalent, suppose $S-ABC$ to be the greater." We neglected to refer to Wentworth's demonstration until after the criticism referred to had been published.

We find, after examination, a number of recently revised geometries containing fallacious demonstrations to the proposition in question.

Prof. Whitaker referring to the above criticism says, "I would state that the proof of Proposition XXIV, Bk. V., in Wentworth's is also incorrect." If the proposition referred to by Prof. Whitaker, is Prop. XX., Bk. V., Wentworth's Revised Edition, 1891, we would say that the fallacy has been corrected.

B. F. FINKEL.

EDITORIALS.

The subscription list to the MONTHLY is growing nicely but it is yet many hundreds short of what it should be.

Occasionally we have sample copies returned to us, with the remark, "It is too fine for my blood." Now it occures to the editors that something of the nature of the MONTHLY is needed to give a better tone to the "blood." Try the MONTHLY a year and see if the tone and vigor of the mind does not improve because of the increased activity of the "blood."

Every lover of Mathematics and every teacher of Mathematics in the United States should take the MONTHLY. Articles are now being published that will after while be reprinted and bound in book form and which alone will then cost the price of the Journal.

Because of the lengthy Biographical sketch published in this number, we have been compelled to defer many interesting communications. It is not the purpose of the MONTHLY to publish such extended biographies, except when, as it is in this case, the life of the man treated justifies it.

We have a number of criticisms and replies to certain problems and articles which will appear as we find room. We hope that our contributors will take these criticisms in a spirit of kindness It is not the purpose of the MONTHLY to publish lampoons, or sarcastic criticisms. Its object is to promote the science of Mathematics and Mathematical teaching.

It is objected that some of the problems published are not worthy of a place in the MONTHLY, being too easy. On the other hand we are admonished not to soar entirely in the higher regions of Mathematical thought but publish problems for the use of classes in the various branches of Mathematics. Now as we are publishing an unusually large Magazine, at an unusually low price, we will try to harmonize these jarring interests by giving space alike to elementary and higher Mathematics.

Dr. Paul Staeckel of the University of Halle and Professor Friedrich Engel of the University of Leipzig will publish through Teubner in Leipzig within a year, under the title "Theorie der Parallellinien," the First Book of Saccheri's marvelous treatise now appearing in English in the AME ICAN MATHEMATICAL MONTHLY. (See Dr. Halsted's Non-Euclidean Geometry, Historical and Expository).

It is a noteworthy honor to America and to the American Mathematical Monthly, thus to have unconsciously anticipated and forestalled the great German scientists.

BOOKS AND PERIODICALS.

A Treatise on Plane and Spherical Trigonometry and Its Application to Astronomy and Geodesy with Numerous Examples. By Edward A. Bowser, LL. D., Professor of Mathematics and Engineering in Rutgers College. 8 vo., cloth and leather back, 368 pp. Price $1.60. Boston: D. C. Heath & Co.

Excepting Chauvenet's, this is the most complete Treatise on Trigonometry

published in America, and in point of excellence it is superior to that work. In the method of treatment, arrangement, typographical execution, and numerous and well selected exercises it has no superior. The definitions of the functions are given "once for all" and need not be restated and modified when obtuse and reflex angles are considered.

In the development of the theoretical part of the subject, the work is especially interesting and clear.

From the beginning the student is carried along with enthusiasm and with the assurance that he is mastering the subject. The unusually large and well chosen collection of problems are suited to every requirement, and by solving these the student learns to do by doing.

The treatment of Trigonometric Elimination, De Moivre's Theorem, Summation of Series, etc., is more complete than is usually given in text-books.

These observations have been gathered by using the book in the class-room.

B. F. F.

A Treatise on Hydrostatics. By Alfred George Greenhill, Professor of Mathematics in the Artillery College, Woolwich, England. 8vo. cloth, 536 pp. Price, $1.90. New York: Macmillan & Co.

The aim of the present Treatise on Hydrostatics is to develop the subject from the outset by means of illustrations of existing problems, chosen in general on as large a scale as possible, and carried out to their numerical results; in this way it is hoped that the student will acquire a real working knowledge of the subject, while at the same time the book will prove useful to practical engineers. PREFACE.

The condensed notation proposed by M. Hospitalier at the International Congress of Electricians of 1891 has been adopted. The gravitation unit of force has been universally employed, except a few problems of cosmopolitan interest. Free use is also made of svmbols, and operations of the Calculus, the author believing "it is easier to learn the Differential Calculus, than to follow a demonstration which attempts to avoid its use."

Particular attention is given to the applications of the subject in Naval Architicture.

The diagrams used to illustrate objects are accurate and attractive.

The book is written in large type and is the best work on the subject that we have yet examined.

B. F. F.

Analytical Trigonometry. Part II. By S. L. Loney, M. A., Late Fellow of Sidney Sussex College, Cambridge, Professor at the Royal Holloway College, 1894. 8vo, cloth, XXVI+(295 to 480) pp. Price, $1.00. New York: Macmillan & Co.

In this text is treated nearly every subject in the modern domain of analytical trigonometry.

It begins with a treatment of Exponential & Logarithmic Series. On page 297 is proof of the inammensurability of $e=2.7182818285\ldots$.

Some of the important subjects discussed in this book are Complex Quantities; De Moivre's Theorem; Circular Functions of Complex Angles; Hyperbolic Functions; Inverse Circular and Hyperbolic Functions; Logarithms of Complex Quantities; Gegory's Series; Principle of Proportional Parts; Errors of Observation; Solution of Cubic Equations; and Maximum and Minimum Values.

The typography and mechanical execution of the book is first class. It is to be hoped that the author will immediately follow it with an equally exhaustive treatise on Spherical Trigonometry. B. F. F.

Elements of Solid Geometry. By Arthur Latham Baker, Ph. D., Prof. of Mathematics in the University of Rochester. 136 pp. 1893. Boston: Ginn & Company.

This compact little volume makes a favorable impression. We note as special features an improved notation, special regard to the perspective of the figures in the diagrams, and the clear presentation of the different parts of the discussions under distinct headings. We are particularly pleased with the importance given to generalized conceptions, the general theorems for the frustum of a pyramid, being first worked out, and then the pyramid, cone, prism, and cylinder being discussed as special cases. Great condensation of matter, as well as a broader conception of the subject on the part of the student, is thus secured. The author does not hesitate to speak emphatically of what he regards as good features in his book. Witness the following: "The whilom popular idea that each proposition must occupy an entire page or pages is discarded. A *short* demonstration is made *short*. The student is not deceived into thinking he has learned a page of geometrical truth, when in fact he has learned but a few lines."

The book closes with a short geometrical treatment of the conic sections.

J. M. C.

The Science Absolute of Space. Independent of the Truth or Falsity of Euclid's Axiom XI (which never can be established *a priori*). By John Bolyai. Translated into English by George Bruce Halsted, A. M., Ph. D., ex-Fellow of Princeton College and Johns Hopkins University, Professor of Mathemetics in the University of Texas. First edition, 1891. Second edition, 1893. Price, bound and postpaid, $1.25.

This translation is prefaced by a valuable Introduction by the translator, in which are revealed some very fine historic facts pertaining to the respective discussionsof Bolyai, Gauss, and Riemann.

Appendix I. Remarks on the preceding Memoir, by Wolfgang Bolyai; Appendix II. Some points in John Bolyai's Appendix compared with Lobtschewsky, by Wolfgang Bolyai. Appendix III. Light from non-Euclidean Spaces on the Teaching of Elementary Geometry, by G. B. Halsted.

This excellent translation should be in the hands of every teacher of geometry.

B. F. F.

About Square Numbers Whose Sum is a Square Number. By Artemas Martin, LL. D., Washington, D. C. 1893.

The above is a pamphlet *reprint* of three articles which appeared in late numbers of the *Mathematical Magazine*, and fills 24 pages of the size of that periodical. A study like this by Dr. Martin, upon a subject with which he is so well acquainted, assures a most interesting and instructive treatment. Those of our subscribers who are fond of the Diophantine Analysis will find a special interest in these pages by the distinguished editor of the *Magazine* and the *Visitor*. J. M. C.

Standard Arithmetic. By William J. Milne, Ph. D., LL. D., President of the New York State Normal College, Albany, N. Y. 430 pp. Price, 65 cts. New York: American Book Company,

The book before us embraces a complete course for Schools and Academies, and is certainly one of the best Arithmetics now before the public. In view of the recent discussion in the MONTHLY, it is interesting to note section 83, p. 68, (1), "The dividend and divisor must be like numbers;" (2), "The quotient must be an abstract number." However, the author fails to show how $\frac{1}{2}$ of \$12 can be found, and in denominate *numbers* calls bu. pk.. etc., a *quotient!*"

The order and arrangement of subjects is the best we have seen, except the placing of common *before* decimal fractions. We regard the position of tables in the back of the book as a nuisance.

There are a great many problems, but in some parts we note lack of variety, and a tendency to state many of them in the direct form, so that after the first is solved the solution of the others becomes only a mechanical process. In a few places,-e.g, in reducing mixed numbers to improper fractions-, the rule given is not derived from the process. In general the explanations are lucid, the steps logical, and the definitions brief and accurate. The treatment of "business arithmetic" is unsurpassed, and as a whole the book so well meets the needs of both teacher and pupil, that we regard it as one of the best approachs to an ideal Arithmetic. "*Elements of Arithmetic*" is the introductory book of the series. J. M. C.

The Educational Times (London) for May is at hand. The list of problems proposed for solution contains five problems reproduced from No. 1 of the MONTHLY.

The last numbers of *Annals of Mathematics* has the following articles; "On the Jacobian Elliptic Functions," by Prof. Irving Stringham; and "Transformation Groups," by J. M. Page. Three exercises are solved and three proposed.

Miscellaneous Notes and Queries. We have received the April and May numbers of this valuable Monthly. We note in the former number an interesting article on "Method of Finding the Date of Easter," by our contributor, Prof. H. A. Wood, A. M., of the Stevens School, Hoboken, N. J., and "Some Practical Geometry," by our subscriber, Thos. P. Stowell, of Rochester, N. Y.

ERRATA.

Page 75, 11th line from bottom, for primative read primitive.

Page 122, last line, for $=\overline{x}$ read $\overline{x}=$.

Page 125, middle of page, for "to $\frac{m+1}{2}$ factors...." read $\frac{m-1}{2}$ factors....

Page 129, in solution to 10, for, From (1) subtract (2), (3), (4), read Subtract (1) from (2), (3), (4).

Page 138, middle of page, for "asperations" read *aspirations*.

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